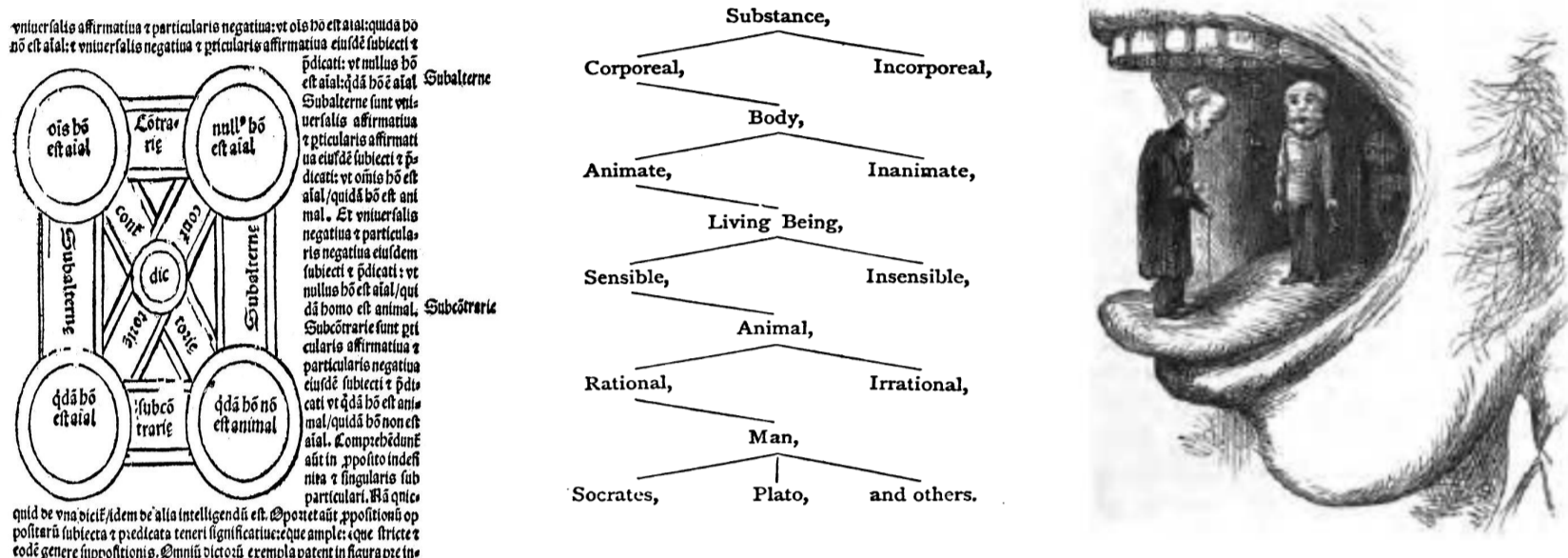# HOW DO SPACES REPRESENT CLASSES?

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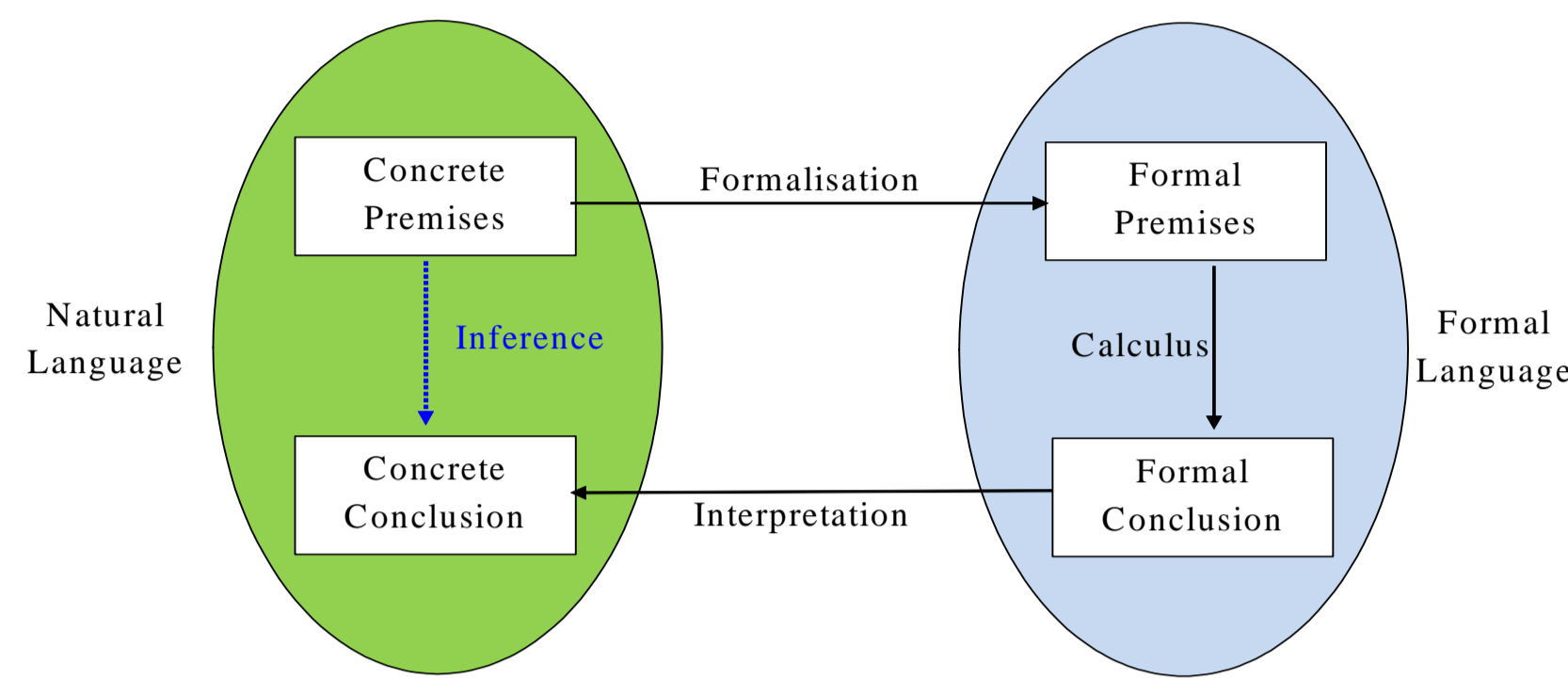
Diagrams in logic

- The use of diagrams in logic is old and common
- Illustrative and pedagogical functions
- Exemples:
  - Square of opposition
  - Tree of Porphyry
  - 'Logic pays Thought a visit'
- What about inference?

Spatial diagrams

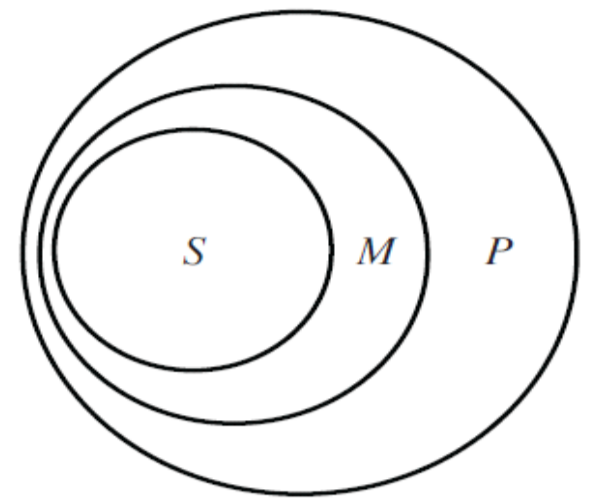Inference is the the business of logic

Calculus can be done with symbols, diagrams and machines
Diagrams used for logic inference are called 'analytic diagrams'
Spatial diagrams are analytic diagrams

Spatial diagrams use spaces to represent classes

For instance, let's consider syllogism:

All S are M
All M are P
All S are P

We represent classes S, M, P with circles.
Premises are depicted with a diagram where circle S is included in circle M, and circle M is included in circle P
The diagram shows that circle S will necessarily fall within circle P

How does a circle represent a class?

Let us now endeavor to seize upon the spirit and characteristic of this system of graphs, and to estimate its value. Its beauty – a violent inappropriate word, yet apparently the best there is to express the satisfactoriness of it upon mere contemplation – and its other merits, which are fairly considerable, spring from its being veridically iconic, naturally analogous to the thing represented, and not a creation of convention.

[C. S. Peirce, CP4, 1933]
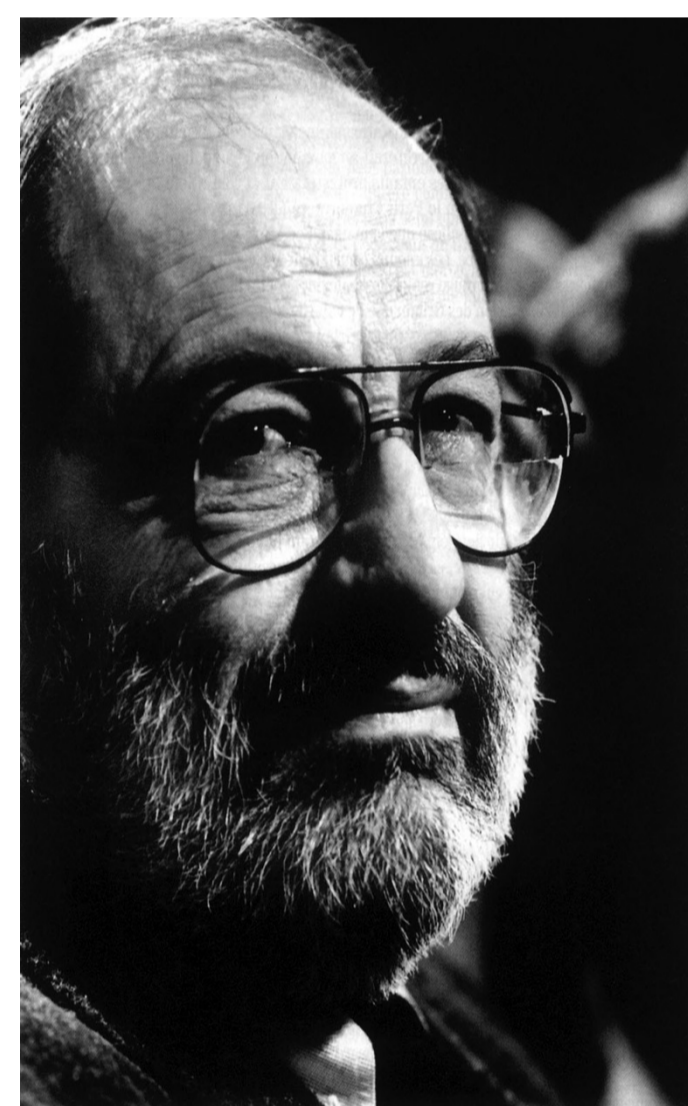
Peirce writes that the beauty of these diagrams comes from their « true iconic » status (4.368), an expression that might suggest that the spatial configuration of the diagrams *imitates* a real spatial situation. If it was so, Peirce's iconism would be truly ingenious; because even if diagrams do show visually interiority and exteriority relations, that doesn't mean that those spatial properties are icons of other spatial properties! The fact of being on not being passionate is not a matter of space. It is at most, to speak in traditional logic words, a matter of possessing or not possessing a given propriety. Why does traditional logic translate this possession and non-possession in terms of belonging and non-belonging to a class? By pure convention […]
But is belonging to a class a spatial fact ? Sure it is not, except the fact that I might be defined to belong to the class of all those who are located in a certain place; but if I belong to the class of those who are passionate, this class would be an abstraction, not a space. Why, in the representation with circles, does the class become a space? By pure convention.

[Eco, *Segno*, 1973]

Is the iconicity of spatial diagrams conventional?
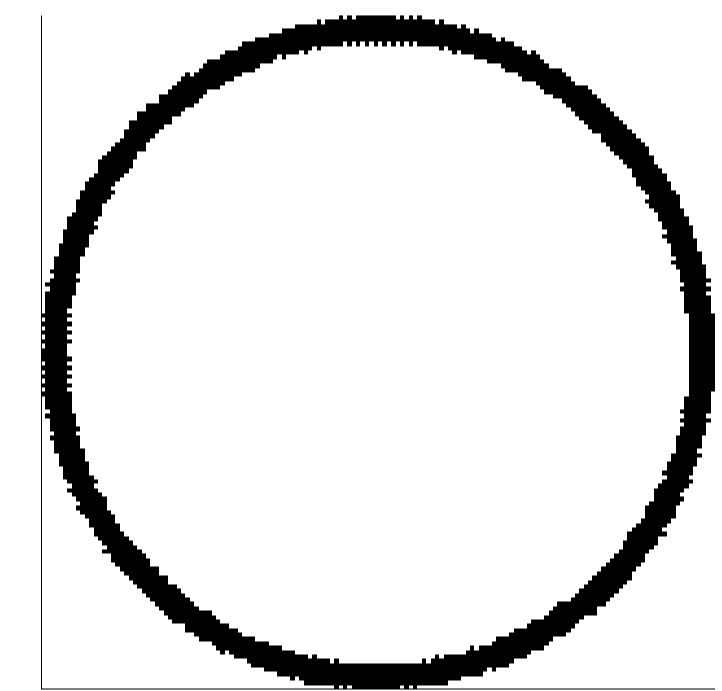
- What is a circle?
- What is a class?
- How does a circle represent a class?

What is a circle?

Euclid's *Elements*, Book I, Defs. 15 & 16:
A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another; And the point is called the centre *of the circle.*
Book III, Prop. 2: *If on the circumference of a circle two points be taken at random, the straight line joining the points will fall within the circle.*
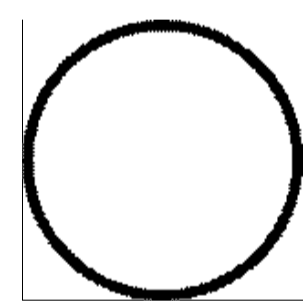

- The circle is not the circular line.
- The circle is the space within the circular line
- The circular line (called circumference) is the border of a territory. The circle is the territory.

What is a class?

- A Class is a collection of things
- A class is the extension of the concept:
  - Concept I = Iberian country
  - Extension I = {Spain, Portugal}
- Different concepts might have the same extension, but a single concept cannot have different extensions

Eco: '*The fact of being on not being passionate is not a matter of space. It is at most, to speak in traditional logic words, a matter of possessing or not possessing a given propriety.*' AGREE
Eco: '*Why does traditional logic translate this possession and non-possession in terms of belonging and non-belonging to a class? By pure convention*' DISAGREE
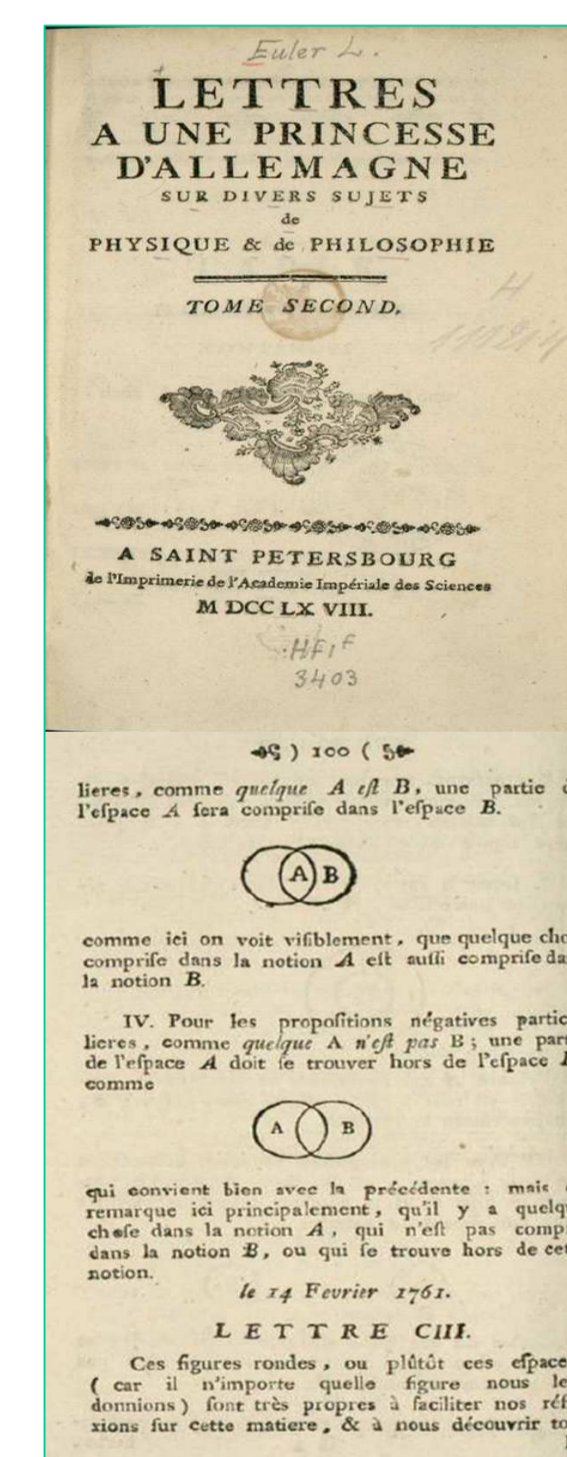
- Spatial diagrams have been popularised by Leonhard Euler in his *Letters to a German Princess* (1768)
- How do circles represent classes?

'As a general notion contains an infinite number of individual objects, we may consider it as a space in which they are all contained. Thus for the notion of *man* we form a space
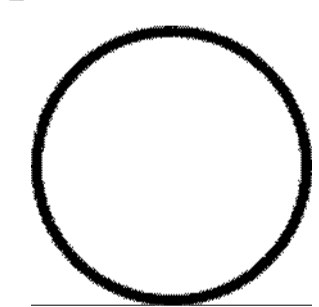
In which we conceive all men to be comprehended'

## The iconicity of spatial diagrams

- Classes are formed by gathering individuals
- The actual existence of the individuals does not matter
- Classes are represented by gathering the individuals in a single space

Eco: 'But is belonging to a class a spatial fact ? Sure it is not, except the fact that I might be defined to belong to the class of all those who are located in a certain place.' PRECISELY.
- Eco's exception is actually the very principle of Spatial diagrams

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- Does 'the spatial configuration of the diagrams imitate a real spatial situation'? YES, the iconicity is not conventional
- Example: "All Iberian countries are European Countries"
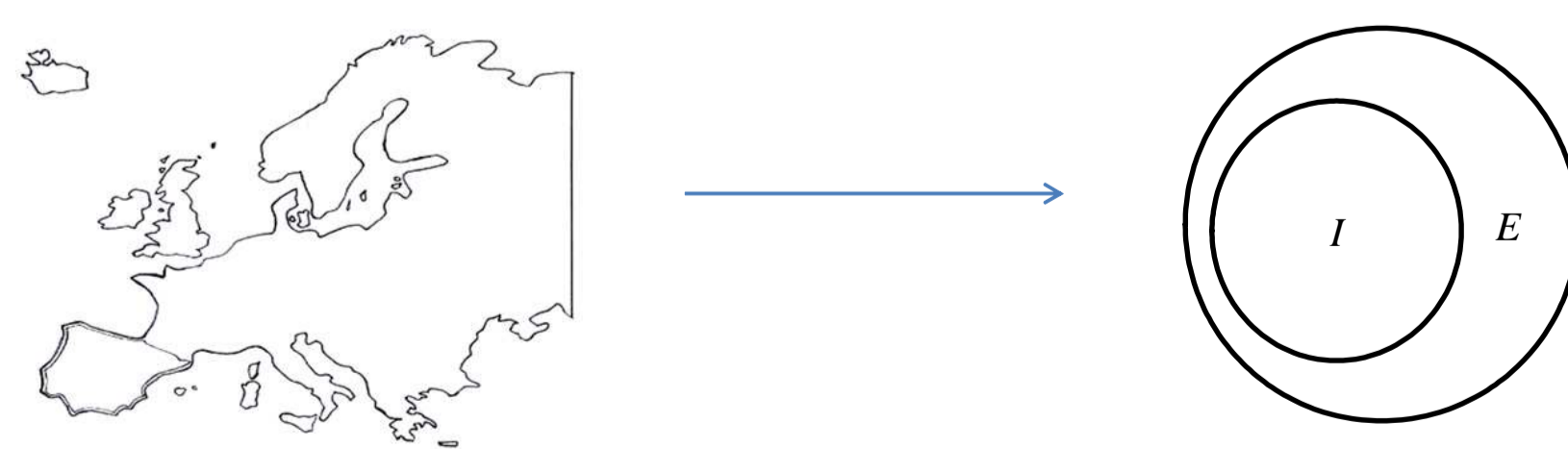
- In spatial diagrams, the relation is not represented. The relation IS.
- The Spatial situation is made by the mental gathering of the individuals in spaces.
- The actual feasibility of the gathering does not matter
- The shape and size of the spaces do not matter

References

- Umberto Eco, *Il segno*, Milano: Mondadori, 1980.
- Leonhard Euler, *Lettres à une Princesse d'Allemagne*, vol. 2, Saint-Pétersbourg: Imprimerie de l'Académie Impériale des Sciences, 1768.
- Amirouche Moktefi & Soo-Jin Shin, "A history of logic diagrams", *in* Dov M. Gabbay, Francis J. Pelletier & John Woods (eds.), *Logic: A History of its Central Concepts*, Amsterdam: North-Holland, 2012, p. 611-682.
- Charles S. Peirce, *Collected Papers*, vol. 4, Cambridge, MA.: Harvard University Press, 1933.
- John Venn, "On the diagrammatic and mechanical representation of propositions and reasonings", *Philosophical Magazine*, vol. 10, 59, 1880, pp. 1-18.

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