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Visualization and the Limits of Scientific Realism

The thesis I will formulate in this talk is that the demarcation line beyond which we should conceive of scientific theories not as true descriptions of the world, but as mathematical instruments enabling us to arrive at correct practical predictions, is not the much-discussed observable/non-observable border (for Mach and the logical positivists blending into the demarcation line between science and metaphysics), but rather the border between, on the one hand, what we can *imagine*, in the sense of being able to form perceptual images, and, on the other hand, what we cannot describe but in abstract symbolic terms. My talk is divided into three sections. The first two, rather brief, sections – under the headings "Meeting Rorty", and "Images of Sellars" – are meant to set the stage, in the form of some personal reminiscences and reflections, for my main argument which I will present in the third, somewhat longer, section: "Believe What You Can Visualize".

Meeting Rorty

It was late in both our lives that I became personally acquainted with Rorty. I met him for the first and the last time in 2004, on two consecutive days. On May 5 I picked him up, with his wife, at the railway station in Budapest where they arrived from a visit in Pécs. I drove them to their hotel and we discussed some organizational details in connection with the talk he was to give on the next day at the Hungarian Academy of Sciences. He seemed tired; we soon parted. I vividly remember the following morning. There was still some time before his talk was due, the sun was shining beautifully; we walked a short distance from the Academy main building to the Danube – to the Chain Bridge – and suddenly I found myself asking him a question. What did he think, I asked, about the *pictorial turn* underway in philosophy? Clearly, this was a rather extraordinary question to put to the man whose name had been, ever since the mid-1960s, closely associated with the term "linguistic turn"[1], and whose 1979 book *Philosophy and the Mirror of Nature* was a single extended attack on "ocular" or "visual" metaphors in philosophy[2] – on the "spectator theory of knowledge"[3]. But wasn't W. J. T. Mitchell's 1992 paper "The Pictorial Turn" directly addressing Rorty's work[4], and didn't the latter by 1990 regard the issues pertaining to linguistic philosophy as having become quaint? Rorty's reac-

[1] The term itself Rorty attributes to Hugo Bergmann, see the editor's "Introduction" in Richard Rorty (ed.), *The Linguistic Turn: Recent Essays in Philosophical Method*, Chicago: The University of Chicago Press, 1967, p. 9.
[2] Richard Rorty, *Philosophy and the Mirror of Nature*, Oxford: Basil Blackwell, 1979, *passim*, but see esp. pp. 11, 39 and 371.
[3] *Ibid*., p. 41. The expectation that "the traditional 'spectatorial' account of knowledge" might soon be "overthrown" is already voiced by Rorty in *The Linguistic Turn*, see his "Introduction", p. 39.
[4] See W. J. T. Mitchell, *Picture Theory*, Chicago: The University of Chicago Press, 1994, p. 11.

tion, there and then, was embarrassing: he has never heard about the expression "pictorial turn", could not imagine what it might mean, and was utterly taken aback by my hurried attempt at some rudimentary explanation. Still, the subject came up again later in the day, during the dinner to which I invited the couple at a restaurant in my home village on the Danube Bend. I think I tried to say something about the implications, for philosophy, of the imagery debate in cognitive science, and about how the ease of accessing and indeed producing pictures in the new digital medium affects not only the ways we communicate, but also the ways we think. This time Dick became interested, as did, also, Mary; they were empathetic, inspiring, and of course absolutely charming; we decided that we should stay in touch and continue discussing the topic.

It did not come to pass. Nor was there an occasion left for me to compare notes with Rorty on the three philosophers who, if I may express it this way, were common heroes to us. I am referring to Heidegger, Wittgenstein, and Wilfrid Sellars, and it is clear that Rorty and I came to hold widely diverging views on them. For the author of *Philosophy and the Mirror of Nature*, Heidegger was, first and foremost, a foe of "the notion of knowledge as accurate representation",[5] a philosopher whose concern was "to explore the way in which the West became obsessed with the notion of our primary relation to objects as analogous to visual perception".[6] My impression is that this dimension in Heidegger's thought never lost its primary significance for Rorty. By contrast, I came to regard the Heidegger of the 1920s as someone who has something fundamental to say about our encounter with the world, and, not incidentally, about our encounter with the visual world. It is in his *Kant and the Problem of Metaphysics* that Heidegger faces the problem of how to reconcile the conceptual with the perceptual. The "power of imagination" – the Kantian *Einbildungskraft* – "refers to all representing in the broadest sense which is not in accordance with perception: conceiving of something, … devising, having an inspiration".[7] As Heidegger puts it, "the correct understanding of the sensible character of the power of imagination" must go hand in hand with an "insight into the primary representational character of thinking".[8] Heidegger not only emphasizes that the power of imagination is a faculty which actually provides *images*,[9] but offers, in a nutshell, a brilliant analysis of the fundamental questions of pictorial representation: of what *likeness* is, and how *general images* are possible.[10]

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[5] *Philosophy and the Mirror of Nature*, p. 6.
[6] *Ibid*., pp. 162 f.
[7] Martin Heidegger, *Kant and the Problem of Metaphysics* (1929), transl. by Richard Taft, Bloomington: Indiana University Press, 1997, p. 91.
[8] *Ibid*., p. 103.
[9] *Ibid*., p. 91.
[10] The following lines can perhaps convey the flavour of Heidegger's analyses here: "It is possible to produce a copy (photograph) … from … a likeness, [a photograph] of a death mask for example. The copy can now directly copy the likeness and thus reveal the 'image' (the immediate look) of the deceased himself. The photograph of the death mask, as copy of a likeness, is itself an image – but this is only because it gives the 'image' of the dead person, shows how the dead person appears, or rather how it appeared. … – Now the photograph, however, can also show how something like a death mask appears in general. In turn, the death mask can show in general how something like the face of a dead human being appears. But an individual corpse itself can also show this. And similarly, the mask itself can also show how a death mask in general appears, just as a photograph shows not only how what is photographed, but also how a photograph in general, appears", *ibid*., p. 66. I have corrected a misprint or mistranslation in the edition here

Heidegger's book on Kant, and especially the passages I refer to here, have never been in the limelight. It is understandable that Rorty did not form a picture of Heidegger the philosopher of images. It is similarly understandable that he was unaware of the later Wittgenstein's preoccupation with pictorial representation. As Rorty put it in *Philosophy and the Mirror of Nature*: "you can't recognize a picture of X *as* a picture of X without being familiar with the relevant pictorial conventions".[11] In the heyday of linguistic philosophy, the later Wittgenstein was invariably read through Goodman's eyes. The uncontested view was that images do not depict, do not resemble; they *denote* – just like the words of verbal language. And *what* they denote will be determined by rules we have to learn. Now this is not at all a view Wittgenstein uniformly entertained. For instance, in the so-called "Part II" of the *Philosophical Investigations*, he outlines cases where understanding a picture appears to be entirely independent of language use. Giving the example of a "picture-face", he remarks: "In some respects I stand towards it as I do towards a human face. I can study its expression, can react to it as to the expression of the human face. A child can talk to picture-men or picture-animals, can treat them as it treats dolls." Let me note that remarks such as this were definitely rare in Wittgenstein's printed works, as available from the 1950s to the 1990s. The printed corpus only partially conveyed the richness, complexities, continuities of, and changes in, Wittgenstein's ideas on pictorial representation. It was only with the publication of the Bergen electronic edition, making his full *Nachlaß* available, that the extent of Wittgenstein's commitment to the idea of images and words playing intertwining roles became clear.[12]

Images of Sellars

In the "Introduction" to his volume *The Linguistic Turn*, Rorty outlines a number of alternatives for the future of philosophy. One of these he characterizes as no longer envisaging "the dissolution of philosophical problems, but rather the creation of new, interesting and fruitful ways of thinking about things in general". On this alternative, "[p]hilosophers would be, as they have traditionally been supposed to be, men who gave one a *Weltanschauung* – in Sellars' phrase, a way of 'understanding how things in the broadest possible sense of the term hang together in the broadest possible sense of the term'."[13] The passage Rorty here quotes, from Sellars' "Philosophy and the Scientific Image of Man", played, way back in the late 1960s, a formative role in the development of my own thinking.[14] Sellars was my first, and most important, mentor in philosophy. We never met in person – in those days Hungarians were seldom permitted to leave the country for a scholarly visit to the States – but we corresponded, and he lavishly fur-

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quoted: the phrase "The copy can now directly copy the likeness" there has "only" instead of "now" (i.e. "nur" instead of "nun").

[11] *Philosophy and the Mirror of Nature*, p. 25.

[12] See my "Wittgenstein's Philosophy of Pictures" (2001), in A. Pichler and S. Säätelä (eds.), *Wittgenstein: The Philosopher and his Works*, Frankfurt a.M.: ontos verlag 2006, pp. 322–353.

[13] *The Linguistic Turn*, p. 34.

[14] The passage in full: "The aim of philosophy, abstractly formulated, is to understand how things in the broadest possible sense of the term hang together in the broadest possible sense of the term. Under 'things in the broadest possible sense' I include such radically different items as not only 'cabbages and kings', but numbers and duties, possibilities and finger snaps, aesthetic experience and death." (Wilfrid Sellars, *Science, Perception and Reality*, London: Routledge & Kegan Paul, 1963, p. 1.)

nished me with preprints and offprints. What I was most impressed by was the particular variety of scientific realism Sellars stood for: the view that science is "continuous with common sense", and the idea of theoretical entities as postulated but real. I am still fascinated by this idea. During my recent, rudimentary, attempts to come to grips with some notoriously difficult issues in the philosophy of time, I found Sellars' suggestion that "time has the status of a quasi-theoretical entity"[15] particularly helpful.[16] Now Sellars the scientific realist stresses that it is of course physics, or rather the future advance of physics, and not metaphysics, that ultimately determines what the nature of the theoretical entity *time* is.[17] In the final section of my talk, I will explain why I think that on this point I have to diverge from Sellars – why I believe that we need something like descriptive metaphysics here to defend the rights of common-sense realism in the face of an excessive scientific realism. As I indicated by way of introduction, my argument will turn on the role of *images* in our thinking.

Sellars does not allow for such a role. What he tells us in "Philosophy and the Scientific Image of Man" is that "all attempts to construe thoughts as complex patterns of images have failed, and, as we know, were bound to fail"[18], that "association of thoughts is not association of images"[19], and that "however intimately conceptual thinking is related to sensations and images, it cannot be equated with them, nor with complexes consisting of them"[20]. But Sellars does not only not *equate* thoughts with images, he actually *excludes* the latter from the realm of the former. As it becomes clear e.g. from his major essay "Empiricism and the Philosophy of Mind", mental episodes, for him, are *linguistic* episodes, and imagery boils down to *verbal* imagery[21] – while at same time, in that very essay, he develops a theory within the framework of which he could easily have explained the status of mental images. According to this theory, thoughts are theoretical entities construed, in primordial times, on the analogy of overt verbal episodes. Sellars does find a place in his framework for *impressions* – but not for images.[22] Experts on Sellars might respond by pointing out that, still, the notion of "picturing" played a central

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[15] Wilfrid Sellars, "Time and the World Order", in Herbert Feigl and Grover Maxwell (eds.), *Minnesota Studies in the Philosophy of Science*, vol. III, Minneapolis: University of Minnesota Press, p. 551.

[16] For a first experiment of mine along Sellarsian lines see my "Time and Communication", in F. Stadler and M. Stöltzner (eds.), *Time and History / Zeit und Geschichte*, Frankfurt/M.: ontos verlag, 2006, pp. 302 f.

[17] "Time and the World Order", p. 593.

[18] *Science, Perception and Reality*, p. 15.

[19] *Ibid*., p. 16.

[20] *Ibid*., p. 32. I find the way this last passage begins telling: "one scarcely needs to point out these days that however intimately conceptual thinking is related to sensations and images, it cannot be equated with them…".

[21] First published in 1956, repr. in *Science, Perception and Reality*, pp. 177 f.

[22] A cognitive psychological theory along what can be regarded as Sellarsian lines was developed in Allan Paivio's *Imagery and Verbal Processes* (New York: Holt, Rinehart and Winston, 1971), one the first contributions to the so-called imagery debate. "Mental images", wrote Paivio, belong to the order of "postulated processes", they are "theoretical constructs", "inferential concepts", i.e. entities or processes themselves not observable, but having observable aspects and implications. Introspective experiencing of visual images on the one hand, and the objective recording of neural phenomena on the other, are empirical observations of a very different sort, but they refer to one and the same theoretical construct of a "mental image". Paivio contrasts his own methodology with "the classical approach to imagery" in which "the term image was used to refer to consciously-experienced mental processes" (*Imagery and Verbal Processes*, pp. 6–11). I will come back to Paivio later in the present talk.

role in his paper "Truth and 'Correspondence' ", or indeed in the chapter on "Picturing" in his book *Science and Metaphysics*. Recall however, that for Sellars picturing was but a relation between configurations of objects in the world on the one hand, and *linguistic* configurations on the other.[23] From his reminiscences of Wittgenstein's *Tractatus* the message of paragraphs 4.016 and 4.02 is completely missing. And this is what Wittgenstein wrote there: "In order to understand the essence of the proposition, consider hieroglyphic writing, which pictures the facts it describes. – And from it came the alphabet without the essence of representation being lost. – This we see from the fact that we understand the sense of the propositional sign, without having had it explained to us."

All this is striking, for Sellars definitely had a sense for images and pictures. As becomes clear when looking at the posthumous volume *Kant and Pre-Kantian Themes: Lectures by Wilfrid Sellars*,[24] in class he loved to draw pictures and diagrams as a means to explain philosophical problems. And the situation becomes really baffling when we realize that in the mid-1930s, when Sellars was studying philosophy at Oxford, his tutor was H. H. Price,[25] whose 1953 book *Thinking and Experience* is without doubt the fundamental twentieth-century philosophical treatise on the role of mental images. By way of ending the present section of my talk, let me quote a passage from that book. "After listening to a lecture on Imageless Thinking", recounts Price, "a lady in the audience came up to the lecturer and said with a puzzled air, 'But, Professor, you can *think*, can't you?' ".[26]

Believe What You Can Visualize

A famous figure that no-one assumes could not think is Albert Einstein. Now Einstein was a thoroughly visual thinker. You are of course familiar with those oft-quoted passages, in the Schilpp volume and in the Hadamard book, in which he insisted that in his creative work the role of the perceptual was paramount, while that of the verbal was merely secondary. "The words or the language", Einstein told Hadamard, "as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images... – ... [These] … elements are … of visual and some of muscular type. Conventional words or other signs have to be sought for laboriously only in a secondary stage…".[27] Or the passage from his autobiographical notes: "When ... memory-pictures emerge, this is not yet 'thinking'. And when such pictures form series, each member of which calls forth another, this too is not yet 'thinking'. When, however, a certain picture

[23] The title of Joseph C. Pitt's book *Pictures, Images and Conceptual Change: An Analysis of Wilfrid Sellars' Philosophy of Science* amounts to a practical joke: in this book a "picture" is defined as a "linguistic item intimately tied to the concepts of a matter-of-fact and truth" (Dordrecht: Reidel, 1981, p. 10).

[24] Atascadero, CA: Ridgeview, 2002, ed. by Pedro Amaral.

[25] See Sellars' "Autobiographical Reflections", in Hector-Neri Castaneda, ed., *Action, Knowledge, and Reality: Critical Studies in Honor of Wilfrid Sellars*, Indianapolis: Bobbs-Merrill, 1975, p. 285. Sellars refers to Price in "Empiricism and the Philosophy of Mind", *loc. cit*., p. 162

[26] H. H. Price, *Thinking and Experience*, London: Hutchinson's Universal Library, 1953, p. 234. – For a brief summary of Price's position on images, see my "Pictorial Meaning and Mobile Communication", in Kristóf Nyíri, ed., *Mobile Communication: Essays on Cognition and Community*, Vienna: Passagen Verlag, 2003, pp. 159 f.

[27] Jacques Hadamard, *The Psychology of Invention in the Mathematical Field*, Princeton University Press, 1949, pp. 142 f.

turns up in many such series, then ... it becomes an ordering element for such series, in that it connects series which in themselves are unconnected. Such an element becomes an instrument, a concept."[28] We can assume that the visual thought-experiments through which Einstein used to explain his special theory of relativity represented pretty much the very train of thoughts that, in the first place, led him to his discoveries.

Based upon what he imagined – what he *visualized* – Einstein developed a view of what time really *was*. I do not take the side of Arthur Fine, who in his important essay "The Natural Ontological Attitude" (an essay Rorty seems to have found congenial,[29] and one that refers to Horwich's "semantic realism" as the closest counterpart, in the philosophy of language, to the author's own position[30]) ascribes an instrumentalist position to Einstein's 1905 paper;[31] I concur, rather, with Thomas Sattig's position, according to which "Einstein's original formulation of Special Relativity", as contrasted with the formulation he adopted under the influence of Minkowski, "was metaphysically a theory of ordinary space and time"[32]. Now the view Sattig himself accepts is the one Minkowski had put forward in 1908. As Sattig maintains: "Spacetime points and regions are not just mathematical metaphors; they are among the most fundamental entries in our ontological inventory. The realistic interpretation was adopted by Minkowski … as well as [after 1908] by Einstein".[33]

My present talk is designed to indicate a line of argument which might cast doubt on the reality of Minkowskian spacetime. Thus, at this juncture I shall part ways with Sattig, and join up with Arthur Fine, according to whom "to claim genuine reality for … the four-dimensional space-time manifold" amounts to accepting ideas which "not only … boggle the mind of the average man in the street …, they boggle most contemporary scientific minds as well". As Fine sees the matter, "the majority opinion among working, knowledgeable scientists" is that relativity theory is "a powerful instrument", but is not understood as a genuine foundation for "realist existence and nonexistence claims".[34]

Now why do I believe that the notion of a four-dimensional spacetime must, the great array of brilliant philosophical treatises to the contrary notwithstanding, indeed boggle the mind? I am coming to my main argument. In "Philosophy and the Scientific Image of Man", Sellars wrote: "it is a familiar fact that not everything that can be con-

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[28] P. A. Schilpp, ed., *Albert Einstein: Philosopher-Scientist*, Evanston, IL: The Library of Living Philosophers, Inc., 1949, p. 7.

[29] See his "Pragmatism, Davidson and Truth" (1986), in his *Objectivism, Relativism, and Truth*, Cambridge: Cambridge University Press, 1991.

[30] Fine, "The Natural Ontological Attitude" (1984), repr. in Martin Curd and J. A. Cover, eds., *Philosophy of Science: The Central Issues*, New York: Norton, 1998, p. 1208.

[31] *Ibid*., p. 1194.

[32] Thomas Sattig, *The Language and Reality of Time*, Oxford: Clarendon Press, 2006, p. 44.

[33] *Ibid*.

[34] Fine, *op. cit*., pp. 1194 f. – Marshalling arguments both from the philosophy of science and the philosophy of religion, William Lane Craig, some fifteen years later, takes a similar position: "A good many philosophers of science think of the four-dimensional, geometrical representation of space-time, not realistically, but *instrumentally*, that is to say, as an elegant and handy way of presenting the Special Theory of Relativity or the General Theory of Relativity…" (*Time and Eternity: Exploring God's Relationship to Time*, Wheaton, IL: Crossway Books, 2001, p. 95). Arguing against the notion of a timeless God, Brentano, too, consistently held that the idea that "time is the fourth dimension of space" was, at best, a *harmless fiction.* (Franz Brentano, *Philosophical Investigations on Space, Time and the Continuum*, transl. by Barry Smith, London: Croom Helm, 1988, pp. 94 ff. and 173 ff., dictations from 1915 and 1917.)

ceived can, in the ordinary sense, be imagined".[35] The position I am here defending, representing a tradition from Plato through Hume to Titchener, Bartlett, Arnheim, H. H. Price, and Allan Paivio,[36] maintains that, on the contrary, nothing can be conceived that cannot in the ordinary, albeit very broad, sense be imagined. Or, to put it in slightly less radical terms: scientific propositions which offer no kind of transition to visual imagery should not be taken as descriptions of what there really is. By "transition", I mean something Wittgenstein seems to have meant with *überführen*, when in § 449 of *Philosophical Investigations* he wrote: "Man bedenkt nicht, daß man mit den Worten *rechnet*, operiert, sie mit der Zeit in dies oder jenes Bild überführt", or as Anscombe has it: "We do not realize that we *calculate*, operate, with words, and in the course of time translate them sometimes into one picture, sometimes into another."

In my view, the definitive theory on how words and images hang together is Paivio's *dual coding approach*, first summarized in his 1971 book *Imagery and Verbal Processes*. Paivio notes that while "the developmental studies inspired by Piaget, Bruner, and Werner all involved the assumption that images are specialized for the representation of concrete objects and events, whereas inner speech is functionally useful in dealing with abstract problems, concepts, and relationships", this functional distinction cannot be rigidly maintained, as is indicated by "the apparent development of relatively abstract (schematic) images and concretization of abstract ideas in the form of specific images". What Paivio emphasizes is that, ordinarily, "neither images nor words act as independent processes"; rather, they continually interact.[37] Now if this is what really happens, as I believe it is, then we can conclude that strings of words that do not give rise to a steady flow of images do not, strictly speaking, refer to anything; they might be symbolic devices facilitating inferences, but they in no way mirror the world.

There is an essential connection between the visual on the one hand, and the motor and the tactile on the other. Paivio reports previous research showing that mature imagery incorporates "*the implicit motor components of imitative acts*", and goes on to show that "a motor component (implicit or explicit) appears to be generally characteristic of images of movement, and of the transformations involved in the generation of an integrated figural image or the solution of more complex problems requiring visual thinking. The motor component somehow facilitates the transition from one substantive part of the stream of thought to another."[38] One is reminded of Arnheim's analysis of *descriptive gestures*, "those forerunners of line drawing", in his 1969 book *Visual Thinking*. As he there puts it: "the perceptual qualities of shape and motion are present in the very acts of thinking depicted by the gestures and are in fact the medium in which the thinking itself takes place. These perceptual qualities are not necessarily visual or only visual. In gestures, the kinesthetic experiences of pushing, pulling, advancing, obstructing, are likely to play an important part."[39] One is reminded, also, of John M. Kennedy's 1993 book *Draw-*

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[35] *Loc. cit.*, p. 5.
[36] Compare my "Kritik des reinen Bildes: Anschauung, Begriff, Schema", in H. Lenk and R. Wiehl, eds., *Kant Today / Kant aujourd'hui / Kant heute*, Münster: LIT, 2006, pp. 77 ff., as well as my "Wörter und Bilder", *Humboldt-Nachrichten – Berichte des Humboldt-Vereins Ungarn*, no. 29, December 2007, pp. 30 f. (digitally accessible at http://www.humboldt.hu/HN29/HN29_Woerter_und_Bilder.pdf).
[37] *Imagery and Verbal Processes*, pp. 27 and 32.
[38] Ibid., pp. 30 f.
[39] Rudolf Arnheim, *Visual Thinking*, Berkeley: University of California Press, 1969, pp. 117 f.

ing and the Blind,[40] providing, in fact, an elaborate new theory of visual and tactile perception. One is reminded of neurologist Antonio Damasio's remark that "[w]hen people visualize what they intend to accomplish, an accompanying bodily response makes them feel the reality of their goal".[41] And one is reminded of Hacking's insistence, in his "Experimentation and Scientific Realism", that it is not so much observability, but rather the possibility of *manipulating* objects, which is the guarantee of reality.[42] The lesson I draw from Hacking's paper is that imaginability and tangibility are closely related, and that, hence, imaginability is a likely criterion of reality.

But let me come back to Minkowski. This is how he began his famous talk in Cologne in 1908: "The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."[43] A mere few lines later there comes the step which, from my present perspective, I see as crucial. Minkowski announces that he "will try to visualize the state of things by the graphic method". He embarks on drawing a diagram (three more will follow in the course of his presentation), saying: "With this most valiant piece of chalk I might project upon the blackboard four world-axes." And he immediately adds that understanding the diagram of course requires some abstraction, because of "the number four"; but such a measure of abstraction "is for the mathematician no infliction". By drawing this diagram, he continues, "we obtain, as an image, so to speak, of the everlasting career of the substantial point, a curve in the world, a *world-line*…".[44] The German wording is: "Wir erhalten alsdann als Bild sozusagen für den ewigen Lebenslauf des substantiellen Punktes eine Kurve in der Welt, eine *Weltlinie*…". Now the twist of course is that the "Bild" we get is not an image at all, since – forgive me for spelling out the obvious – a four-dimensional diagram cannot be drawn, cannot be visualized, cannot be imagined.[45] I guess this has been pointed out innumerable times, but let me here just refer to the 1965 paper by Peter Geach, "Some Problems about Time", observing some of the oddities of Minkowski's graphs, and let me quote from Strawson's editorial introduction to the

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[40] New Haven, CT: Yale University Press.

[41] Antonio R. Damasio, *Descartes' Error: Emotion, Reason, and the Human Brain*, New York: Grosset / Putnam, 1994, here quoted after Stefan Klein, *The Secret Pulse of Time: Making Sense of Life's Scarcest Commodity*, New York: Marlowe & Co., 2007, pp. 222 f.

[42] Ian Hacking, "Experimentation and Scientific Realism" (1982), repr. in Curd and Cover, eds., *Philosophy of Science*, p. 1157.

[43] Hermann Minkowski, "Space and Time", in H. A. Lorentz, A. Einstein, H. Minkowski and H. Weyl, *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity* (1923), Mineola, NY: Dover Publications, 1952, p. 75.

[44] *Ibid*., p. 76.

[45] Clearly there exist methods of n-dimensional visualization in mathematics, a spectacular one – and probably the best known – being the parallel coordinates system by Alfred Inselberg. But Inselberg never suggested that his visualizations are as it were depictions of anything in the real world. In a telling introductory passage of his recent book *Visual Multidimensional Geometry and Its Applications*, he referred to the 1917 paper "In What Way Does it Become Manifest in the Fundamental Laws of Physics that Space has Three Dimensions?" by the physicist Paul Ehrenfest – a close friend, incidentally, of Einstein – as showing, say, that "planetary orbits are stable only in space of dimension 3. Higher-dimensional planetary systems, if they ever existed, would have a short career due to the orbits' instability, which offers an interesting hypothesis for the dimensionality of our habitat" (Alfred Inselberg, *Parallel Coordinates: Visual Multidimensional Geometry and Its Applications*, Dordrecht: Springer, 2009, p. 2).

volume in which the Geach paper was included. “Geach edges his common sense with logic”, writes Strawson, “to attack some fanciful theorizing – claiming to derive respectability from physics – which, in place of our ordinary conception of objects undergoing change, advocates thinking of a three-dimensional-object-at-a-time as a ‘temporal slice’ of a four-dimensional object. He presses his criticisms by urging the lack of analogy, the radical differences, between spatial and temporal order.”[46] The Geach–Strawson pair has elicited some angry comments from J. J. C. Smart, who in a paper in 1972 stressed that although “in popular exposition” Minkowski did in fact attempt graphic visualization, “his *argument* is not the analogy with graphs. His argument is that only space-time entities are invariant…”.[47] But this is precisely the point. Minkowski devised but a mathematical instrument, presenting it, however, as a true description of the real world. As Arnheim has put it in his *Visual Thinking*: while grasping the view of time suggested by the special theory of relativity can be supported by visualizing the alternation of the images of two systems, “one for which an object is in motion and another for which the same object is at rest”, the “fourth spatial dimension”, postulated subsequently, is “a purely mathematical construct”, not accessible to our mental imagery.[48]

I will come back to Minkowski and to Smart in a minute, but let me just pause to present two famous passages by another great German mathematician, heir to the Einstein–Minkowski tradition: Hermann Weyl. The first passage is from Weyl’s book *Space–Time–Matter*, originally published in 1918. “[T]he scene of action of reality”, Weyl writes, “is not a three-dimensional Euclidean space, but rather a *four-dimensional world, in which space and time are linked together indissolubly*. However deep the chasm may be that separates the intuitive nature of space from that of time in our experience, nothing of this qualitative difference enters into the objective world which physics endeavours to crystallise out of direct experience. It is a four-dimensional continuum, which is neither ‘time’ nor ‘space’. Only the consciousness that passes on in one portion of this world experiences the detached piece which comes to meet it and passes behind it, as *history*, that is, as a process that is going forward in time and takes place in space.”[49] The second passage is from Weyl’s *Philosophy of Mathematics and Natural Science*, originally published in 1927 in German. As Weyl here puts it: “The objective world sim-

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[46] P. F. Strawson, ed., *Studies in the Philosophy of Thought and Action*, London: Oxford University Press, 1968, p. 5.

[47] J. J. C. Smart, “Space-Time and Individuals”, in Richard Rudner and Israel Scheffler, eds., *Logic & Art: Essays in Honor of Nelson Goodman*, Indianapolis: Bobbs-Merrill, 1972, p. 7. My impression is that Smart here has moved away from the position in his *Philosophy and Scientific Realism*. He there wrote: "many of the puzzles and paradoxes of relativity … can most easily be resolved by drawing diagrams of Minkowski space-time, in which most of [the] at first sight counter-intuitive facts will at once look quite obvious" (London: Routledge & Kegan Paul, 1963, pp. 136 f.).

[48] Arnheim, *op. cit.*, pp. 288–291. “If a fourth spatial dimension cannot be visualized”, Arnheim goes on to write, “it is probably because … [b]eyond [the third dimension] geometrical calculations – just as any other multidimensional calculations, such as factor analysis in psychology – must be content with fragmentary visualization, if any. This also means probably putting up with pieces of understanding rather than obtaining a true grasp of the whole. – No fourth dimension of space, however, is in fact claimed to exist by modern physics. It is, in the words of Arthur Eddington, ‘a fictitious construction’ ” (*ibid.*, p. 292). The piece by Eddington Arnheim refers to is the chapter “Spherical Space” in the former’s *The Expanding Universe* (1933), Arnheim quotes from the collection by Milton K. Munitz, ed., *Theories of the Universe: From Babylonian Myth to Modern Science*, Glencoe, IL: The Free Press, 1957.

[49] Hermann Weyl, *Space–Time–Matter* (4th German edition 1921), Mineola, NY: Dover Publications, 1952, p. 217.

ply *is*, it does not *happen*. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time."[50] What Richard Gale says about this passage does just as well fit the first one, namely that it should be understood as a metaphor, since, if taken literally, it would be simply absurd.[51] But let me make two comments. First, that whether taken literally or not, these passages are metaphysical statements, not implied by the mathematics on which they are apparently based. This is especially conspicuous in the case of the 1927 formulation, following in the book after an extended, partisan philosophical argument. My second comment is that metaphors are meaningless if they cannot be visualized, as Weyl's obviously cannot. I conclude that the Minkowski–Weyl interpretation of space-time is a merely instrumental one. And I suggest that scientific realism must end, and common-sense realism ought to be defended, at the point where mathematics ceases to be backed by images.

Clearly, most of mathematics *is* backed by images. Arnheim himself, in the chapter "Thinking with Pure Shapes" in his *Visual Thinking*, stressed that not only "self-evident geometry", but also arithmetics and algebra have a thoroughly perceptual basis, that "[c]ounting is preceded by the perceptual grasp of groups", and that "[n]umbers are perceptual entities, visual and to some extent tactual and auditory".[52] Recent developments suggest that the 19-century visualization Angst in mathematics, and in the philosophy of mathematics, is receding. In his book *Visual Thinking in Mathematics*, Marcus Giaquinto convincingly argues that it is indeed possible "to achieve generality when thinking with particular images"[53] – a geometrical proof *can*, and when possible, *should*, proceed visually; that "[s]o far from being language based, the origin of our knowledge of simple sums seems to be a kind of finger expertise"[54], and both arithmetics and number theory allow for visual proofs;[55] that in algebra "[s]ubstitution, relocation, copying, deletion, and insertion" – that is, the "major classes of symbol manipulation" – are typically "performed in visual imagination, when moving from one term or formula to another. It is likely that in *some* cases, especially symbol relocation, the visualizing has a motor element";[56] and that even in analysis there is room and need for visualization – Giaquinto refers to, and elucidates, the famous Cambridge mathematician J. E. Littlewood's piece "Post-script on pictures". Littlewood, Giaquinto writes, did indeed believe that "a diagram could provide proof of an analytic theorem".[57]

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[50] Hermann Weyl, *Philosophy of Mathematics and Natural Science*, Princeton, NJ: Princeton University Press, 1949, p. 116.

[51] Richard M. Gale, ed., *The Philosophy of Time: A Collection of Essays* (1967), London: Macmillan, 1968, pp. 298 f.

[52] Arnheim, *op. cit*., pp. 221 f., 211 and 213. On p. 214 Arnheim refers to Marguerite Lehr's highly interesting introduction to Catherine Stern's seminal book, *Children Discover Arithmetic: An Introduction to Structural Arithmetic*, London: George G. Harrap, 1953. Catherine Stern was, at the New School for Social Research from 1940 to 1943, research assistant to Max Wertheimer, the founder of Gestalt Psychology.

[53] Marcus Giaquinto, *Visual Thinking in Mathematics: An Epistemological Study*, Oxford: Oxford University Press, 2007, p. 151.

[54] *Ibid*., p. 123.

[55] On this issue see also Michael D. Resnik, *Mathematics as a Science of Patterns*, Oxford: Clarendon Press, 1997, cf. esp. pp. 229 ff.

[56] Giaquinto, *op. cit.*, p. 203.

[57] *Ibid*., p. 163. – As Littlewood puts it in the section "Post-script to pictures", in the volume *Littlewood's Miscellany* (Cambridge: Cambridge University Press, 1953, repr. 1986, p. 54): "My pupils *will* not use

The point where visualization in mathematics utterly breaks down is where it purports to picture time as a fourth dimension of space. It is here common-sense realism has to step in. To defend common-sense realism involves explaining, without explaining away, some crucial common-sense metaphors. Now this is how J. J. C. Smart begins his 1949 paper "The River of Time": "There are certain metaphors which we commonly feel constrained to use when talking about time. We say that we are advancing through time, from the past into the future, much as a ship advances through the sea into unknown waters. Sometimes, again, we think of ourselves as stationary, watching time go by, just as we may stand on a bridge and watch leaves and sticks float down the stream underneath us. … Thus instead of speaking of our advance through time we often speak of the flow of time. … These metaphorical ways of talking are philosophically important in a way in which most metaphorical locutions are not. They … are, in some way, *natural* to us; at first sight, at any rate, it seems difficult to see how we could avoid them."[58] Difficult or not, Smart did his best to demonstrate the alleged spuriousness of these common-sense metaphors. By contrast, I believe we should strive to build up a philosophical strategy which in fact vindicates them. The sketching of such a strategy would of course be the topic of another talk. Coming to the end of the present one, there remain three questions.

The first, lurking in the background throughout my argument: what does "imaginability" amount to? Is imaginability confined to what we, in our world as it is actually given to us, can in fact imagine? Should we not, rather, say what Reichenbach, referring to Helmholtz, suggests, namely that "imagining … visually" a world different from ours is indeed possible, by "depicting the series of sense perceptions which one would have if one lived in such a world"; and that "human beings, living in a non-Euclidean world, would develop an ability of visualization which would make them regard the laws of non-Euclidean geometry as necessary and self-evident, in the same fashion as the laws of Euclidean geometry appear self-evident to us."[59] My stance here is similar to Ramsey's, commenting on the *Tractatus*: "what we can't say we can't say, and we can't whistle it either". What we can't imagine we can't imagine, and can't whistle it either. We can imagine, for we can visualize, spherical geometry, although it constitutes a kind of non-Euclidean one. But we can in no way visualize, say, spacetime with eleven dimensions as string theory suggests; and here we should not let physicists string us along, but should assume a decidedly instrumentalist attitude.

Secondly, with the views I have here put forward, where would I locate my position in the philosophy of science? I still feel myself belonging to the realist camp, siding with Grover Maxwell's contention that there is a "continuous transition from observabili-

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pictures, even unofficially and when there is no question of expense. This practice is increasing; I have lately discovered that it has existed for 30 years or more, and also why. A heavy warning used to be given that pictures are not rigorous; this has never had its bluff called and has permanently frightened its victims into playing for safety. Some pictures, of course, are not rigorous, but I should say most are (and I use them whenever possible myself)." Littlewood and Wittgenstein were friends. They first met in Manchester, and then again in Cambridge. The greater part of Wittgenstein's numerous drawings in his manuscripts pertain to the foundations of mathematics; and a major message of Wittgenstein's philosophy of mathematics is that mathematical facts and physical facts *overlap*; images of physical facts, then, can indeed convey mathematical truths.

[58] J. J. C. Smart, "The River of Time", *Mind*, vol. 58, no. 232 (Oct. 1949), p. 483.

[59] Hans Reichenbach, "The Philosophical Significance of the Theory of Relativity", in Schilpp, ed., *Albert Einstein*, pp. 300 and 308.

ty to unobservability" which has no relevance at all to the existence/nonexistence issue;[60] siding with Hacking's view that the experimenter is necessarily, and rightly, convinced of the reality of a great many unobservable entities;[61] and of course sharing Sellars' faith in the power of science to draw up ever more correct images of the world. But I also take seriously the cautioning words Sellars again and again voiced: what contemporary science offers consists, in no small measure, of promissory notes. The position I suggest appears to me to offer a felicitous compromise between common-sense realism and scientific realism. But there is one variety of scientific realism I can, clearly, not make friends with: *structural realism*, although this is considered, it seems, by many realists and anti-realists alike as "the most defensible form of scientific realism"[62]. Structural realism says that we should epistemically commit ourselves only to the mathematical or structural content of our theories. I believe that, quite on the contrary, we should commit ourselves to the visualizable content of them.

And so, by way of conclusion, let me now ask myself a third, very brief, and somewhat emotional, question: had that hoped-for next meeting with Rorty happened, how would it have played out? Certainly we would have been in agreement that verbal language in general, and the language of theories in particular, do *not picture*; they are conventional instruments the community of human beings use. But everyday thinking and communication, as well as scientific theories, involve more than just verbal language. They involve images, too. They involve, indeed they fundamentally rely on, visualizations. And can we not say that the images presupposed, or suggested, by our most successful theories amount to something like *mirrors of nature*? I believe Rorty would have found this idea intriguing. He might even have liked it.

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[60] Grover Maxwell, "The Ontological Status of Theoretical Entities" (1962), repr. in Curd and Cover, eds., *Philosophy of Science*, p. 1057.

[61] Hacking, *op. cit.*, pp. 1154 f.

[62] See the opening sentence of James Ladyman's excellent discussion in the *Stanford Encyclopedia of Philosophy* entry "Structural Realism" (http://plato.stanford.edu/entries/structural-realism, first published in Nov. 2007), see also Ladyman's earlier book *Understanding Philosophy of Science*, London: Routledge, 2002, as well as the unpublished, but widely cited, chapter "The Scientific Realism Debate" in Ioannis Votsis' 2004 PhD dissertation.